Experiment Two: Measurement of g using Kater’s Pendulum

Introduction - Compound Pendulum

Kater’s pendulum, that will be used to measure the acceleration of gravity, g, in lab 2 is a special case of the compound pendulum; that is, a rigid body suspended from a point, O, which is free to rotate about an axis through the point 0 as shown in fig. 2. For a displacement of $\theta$ about the point O, we have the usual equation of motion

Fig1. The compound pendulum

$$I_O \frac{d^2 \theta}{dt^2} = mgh_o \sin \theta$$

(1)

If the pendulum is suspended from P we have

$$I_P \frac{d^2 \theta}{dt^2} = mgh_p \sin \theta$$

(2)

Using the parallel axis theorem we get

$$I_O = I_{cm} + mh_o^2 \quad \text{and} \quad I_P = I_{cm} + mh_p^2$$

(3)
For small angles, \( \theta \), we have the usual solutions to eq. 1 and 2 that yield a frequency

\[
\omega_O^2 = \frac{mgh_O}{I_O} \quad \text{and} \quad \omega_P^2 = \frac{mgh_P}{I_P}
\]  

The period for oscillations about the two axis through the points O and P are

\[
T_O = 2\pi \sqrt{\frac{I_O}{mgh_O}} \quad \text{and} \quad T_P = 2\pi \sqrt{\frac{I_P}{mgh_P}}
\]  

We now define the radius of gyration as the length \( k \) such that the moment of inertia of a body about an axis, for example an axis through the center of mass, is given by

\[
I_{cm} = mk^2. \quad \text{We can now write}
\]

\[
I_O = m(k^2 + h_O^2) \quad \text{and} \quad I_P = m(k^2 + h_P^2)
\]  

Substitute into equations 5 we obtain

\[
T_O = 2\pi \sqrt{(k^2 + h_O^2)/gh_O} \quad \text{and} \quad T_P = 2\pi \sqrt{(k^2 + h_P^2)/gh_P}
\]  

For the periods to be equal, \( T_O = T_P \), the condition that

\[
\frac{k^2 + h_O^2}{gh_O} = \frac{k^2 + h_P^2}{gh_P}
\]  

Equation 8 can only be satisfied if \( k^2 = h_O h_P \)

Note that the period in this case is the same as the period of a simple pendulum with a bob of mass \( m \) and length \( h_O + h_P \).

**Experiment 2**

**Determination of g using Kater’s reversible pendulum**

Kater’s pendulum is complex pendulum that was designed by Henry Kater (1777-1835) to measure the acceleration of free fall. It consists of a metal bar with knife edges attached near the ends and two weights that can slide between the knife edges. The bar is pivoted from each knife edge in turn and the positions of the weights are adjusted so that the period of the pendulum is the same with both pivots. The period is then given by the formula for the simple pendulum, which enables g to be calculated to very good precision.
In the laboratory you will use a pendulum fabricated by the Daedalon Corporation that consists of a long steel bar with two knife edges located near the ends. The bar has two brass bobs of unequal mass that can be adjusted in order to find the configuration for which the period of oscillation is the same when the pendulum is suspended from either knife edge, which is also equal to the period of an equivalent simple pendulum of length equal to the distance between the two knife edges.

**Basic Description**

The end with large bob is called A end. The pendulum can be rotated so that A can in turn be up or down. The goal of this experiment will be to determine the relative position of the two bobs such that the measured period is the same for the two knife edges. You will be provided with an electronic timer that has a least count of 0.0001 s. This timer is used to measure the period of one complete cycle of the pendulum. You will also have available calipers and rulers to measure the distance of the bobs from the end of the bar.