Phys 433 – Lecture 6 (Review Error Analysis)

• Propagation of errors in case of dependent variables that are functions of one or more measured variable or combining standard deviations of uncertainties to estimate a resultant uncertainty

• Consider the function \( x = f(u,v,...) \) and its most probable value \( \bar{x} = f(\bar{u},\bar{v},...) \)
  - Individual measurements yield \( x_i = f(u_i,v_i,...) \)
  - In limit of infinite measurements the variance is given by
    \[
    \sigma_x^2 = \lim_{N \to \infty} \frac{1}{N} \sum (x_i - \bar{x}_i)^2
    \]
  - The deviations of \( x_i - x \) are given by
    \[
    x_i - \bar{x} \cong (u_i - \bar{u})\left(\frac{\partial x}{\partial u}\right) + (v_i - \bar{v})\left(\frac{\partial x}{\partial v}\right) + ...
    \]

• Consequently
  \[
  \sigma_x^2 \cong \lim_{N \to \infty} \frac{1}{N} \sum \left[ (u_i - \bar{u})\left(\frac{\partial x}{\partial u}\right) + (v_i - \bar{v})\left(\frac{\partial x}{\partial v}\right) + ... \right]^2
  \]
  \[
  \cong \lim_{N \to \infty} \frac{1}{N} \sum \left[ (u_i - \bar{u})^2\left(\frac{\partial x}{\partial u}\right)^2 + (v_i - \bar{v})^2\left(\frac{\partial x}{\partial v}\right)^2 + 2(u_i - \bar{u})(v_i - \bar{v})\left(\frac{\partial x}{\partial u}\right)\left(\frac{\partial x}{\partial v}\right) + ... \right]
  \]
• The first two terms are simply the variances given by
\[ \sigma_u^2 = \lim_{N \to \infty} \left[ \frac{1}{N} \sum (u_i - \bar{u}_i)^2 \right], \quad \sigma_v^2 = \lim_{N \to \infty} \left[ \frac{1}{N} \sum (v_i - \bar{v}_i)^2 \right]. \]

• The third term is the covariance \( \sigma_{uv}^2 \equiv \lim_{N \to \infty} \left[ \frac{1}{N} \sum [(u_i - \bar{u})(v_i - \bar{v})] \right] \)

• Using these definitions we have the error propagation eq.
\[ \sigma_x^2 \equiv \sigma_u^2 \left( \frac{\partial x}{\partial u} \right)^2 + \sigma_v^2 \left( \frac{\partial x}{\partial v} \right)^2 + \ldots + 2\sigma_{uv}^2 \left( \frac{\partial x}{\partial u} \right) \left( \frac{\partial x}{\partial v} \right) + \ldots \]

• The first two terms are the averages of the squares of the deviations in \( x \) produced by uncertainties in \( u \) and \( v \)
• The cross term, covariance, vanishes if the variables are uncorrelated.
A charged particle other than an electron traveling through matter loses energy by electromagnetic interactions with electrons thereby raising them to higher energy states including continuum states resulting in a free or ionized electron (ionization or atomic excitation).

- The result is that the incident particle loses kinetic energy.
- If the incident particle has sufficiently low energy it will come to rest in the material (KE=0) after traveling a distance called the *Range* of the particle.
- If the incident particle has very high energy ($\beta \sim 1$) it will exit the material with less energy after having undergone many interactions or *multiple coulomb scattering*.
  - Each interaction results in transfer of momentum between the incident particle and the scattering target, electron.
  - The incident particle will exit the material in a direction differing from that of the incident particle by an angle $\theta_{ms}$, the *multiple scattering angle*.
• For very relativistic (non-electrons) charged particles the mean rate of energy loss is given by the Bethe-Bloch equation

\[-\frac{dE}{dx} = K\frac{Z^2}{A}\beta^2 \left[ \frac{1}{2} \ln \left( \frac{2m_e^2\beta^2\gamma^2T_{\max}}{I^2} \right) - \beta^2 - \frac{\delta}{2} \right]\]

- \(T_{\max}\) is max. KE can be given to free electron in a single collision
- \(K = 4\pi NA_0e^2m_ec^2 = 0.307\) MeV-cm²
- \(Z\), atomic mass of absorber
- \(A\) is in g/mol
- \(N_A\) Avogadro's number
- \(m_e\) electron mass = 0.510998 MeV
- \(I\), mean excitation energy in eV
- \(\beta = v/c\) and \(\gamma = [1 - (v/c)^2]^{-1}\)
- \(\delta\), Density effect correction to ionization

• The units of \(dE/dx\) are MeV-g⁻¹-cm²; multiplying by the density, \(\rho\) in g/cm³, gives MeV/cm

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Physics 433 - Lecture 6

- The dE/dx plot shown in the previous slide displays the characteristic rapid fall and low $\beta$ (dominated by the $1/\beta^2$ term) followed by the logarithmic increase as $\beta$ approaches 1
  - As the speed of the incident charged particle increases it's electric field Lorentz contracted along the direction of motion and it's transverse field is increased at large distances by a factor $\gamma$, which results in the logarithmic increase shown on the plot
  - The transverse field is screened by the density effect by the effective polarization of the surrounding molecules, which in turn reduces somewhat the contribution of the logarithmic term
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- From the curve you will note that minimum ionizing particles (mips) typically lose about 1.5 MeV-g⁻¹-cm² in traversing matter
  - This is not surprising given that all atoms but hydrogen have Z/A of about ½ and multiple scattering energy loss occurs by interactions with electrons
  - Hydrogen on the other hand has a Z/A = 1 or twice as many electrons per atom
  - In making quick “back of the envelop” calculations of energy loss it is usual to take a value of about 1.5 MeV-g⁻¹-cm²
- For very slow particles (β < 1) the distance traveled in a material before coming to rest, Range, is a very useful parameter
  - From dE/dx = -f(E) we can obtain dx = dE/f(E) and by integrating dx from E₀ to 0 we obtain the Range of the particle in a given material
The loss of energy by multiple scattering is a statistical process because the energy loss is the result of many, many collisions. The fluctuations about the expected (calculated) energy loss value is called straggling. Similarly, the range of a particle of given energy has a certain range straggling.

We have not discussed energy loss by high electrons because they do not lose most of their energy by ionizing collisions. Electrons are light and when they scatter they receive significant accelerations resulting in the radiation of photons which is called bremsstrahlung.
• Detectors in nuclear and elementary particle physics are all based on the transfer of energy to the detector mass where it is converted to a form that is accessible to detection
  - Charged particles transfer energy mainly through electromagnetic (EM) interactions with atomic electrons that results in excited or ionized atoms
  - Charged particle can also transfer energy by radiating an energetic gamma ray by an EM-interaction in the field of the nucleus - photon exchange.
  - Photons can interact with charge and convert to electron positron pairs that in turn transfer energy
• Almost all modern detectors convert the information into electrical impulses to take advantage of fast electronics and modern computers
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• Considerations for detectors
  - *Sensitivity to radiation* you wish to detect
    • Depends on cross-section, detector mass, and inherent noise in detector
  - *Detector response* – how well the detector preserves the inherent information as the signal is processed
  - *Energy resolution* $\Delta E/E$
  - *Response time* or time it takes to generate the signal after the passage of the radiation
  - *Detector Efficiency* or fraction of total “events” that are observed by the detector—depends on detector geometry (design), noise thresholds etc
  - *Dead time* of detector or how long does it take for the detector to process an event and recover
Modern Collider detectors employ a variety of detector subsystems in order to record the maximum information possible on all radiation emitted from the interaction.

The ATLAS detector being developed for the Large Hadron Collider at CERN is an example.
The generic collider detector has
- Inner tracking charged particle system immersed in a magnetic field
- Electromagnetic calorimeter
- Hadronic calorimeter
- Muon System

All of the above systems depend on Electromagnetic interactions to transfer energy to the detector medium which can then be processed and recorded
- There are many types of detectors; however a very large group of detectors depend on detecting the ionization electrons by collecting the charge that drifts along an applied electric field to an electrode