Experiment 6

Measurement of G – the gravitational constant

Newton’s law of universal gravitation (proposed in 1686) states that the attractive force between two bodies is proportional to the product their masses and inversely proportional to the square of the distance between them. The constant of proportionality, \( G \), is called the universal gravitational constant or simply big \( G \). The first measurements of \( G \) were made in 1798, 100 after Newton’s proposal, by Henry Cavendish and colleagues using a torsion balance. They obtained a value of \( G \) accurate to about 1%.

The torsion balance consists of two masses connected by a rod (dumbbell) suspended by a thin fiber or band. Placing two larger masses on opposite sides of the dumbbell results in an attractive force on the small masses and a twisting of the fiber. The magnitude of the twist allows one to determine \( G \) as described below.

Knowing \( G \) and the acceleration of gravity at the surface of the earth the mass of the earth can be determined. The sun’s mass can be determined from the size and period of the Earth’s orbit around the sun. For this and other reasons physicists have continued to measure and improve the precision of \( G \), usually using improved torsion balances. Some of the most precise measurements of \( G \) have been made at the University of Washington by the Eot-Wash group using a very sophisticated torsion balance. Details of their apparatus can be found at the Eot-Wash web site.

Apparatus

The apparatus you will use in this experiment is shown schematically in Figure 1 consists of two small lead spheres of mass \( m_2 \) mounted at the end of a low mass rod of length \( 2d \). The torsion pendulum or balance is accomplished by suspending the rod, which has a mirror mounted at the center, from a quartz fiber. The small spheres are attracted by two larger lead spheres, mass \( m_1 \), whose center is at a distance \( a \) from the center of the small sphere. When the large masses are brought near the small masses, the force \( F_G = G \frac{m_1 m_2}{a^2} \) results in an angular deflection \( \theta \) which is measured by reflecting a laser beam from the mirror (fig. 2).

The restoring force of the twisted fiber, \( F_R \), balances the gravitational force, \( F_G \).

Our restoring force is \( F_R = -c(d\theta) \). Note that this is the classic harmonic motion equation, so when we displace the system from its equilibrium position it will oscillate with frequency. By measuring the oscillation period of the torsion pendulum we can determine the restoring force, which in turn allows us to determine the gravitational constant \( G \). It is convenient to use torque and moment of inertia to develop the useful relationships.

The torque, \( \tau = I d^2 \theta / dt^2 = 2 F_G d = k\theta \), where \( I = 2m_2 d^2 \) is the moment of inertia and \( k \) is the torsion constant.
The differential equation \[ \frac{d^2 \theta}{dt^2} + \frac{k}{l} \theta \] as you can easily show describes harmonic motion with frequency \( \omega = \frac{2\pi}{T} = \left( \frac{k}{l} \right)^{1/2} \) which leads to \( T = 2\pi \left( \frac{l}{k} \right)^{1/2} \).

From geometry (see Figure 2) \[ 2\theta = \frac{S}{2L} \quad \text{or} \quad \theta = \frac{S}{4L} \]

by appropriate substitution we get

\[ kT^2 = \frac{\tau}{\theta} T^2 = 4\pi T^2 \]

and on further substitution we obtain

\[ G = \frac{\pi^2 b^2 Sd}{mT^2 L} . \]

The equilibrium position is obtained by measuring the furthest most points reached by the light beam and averaging over several cycles as described in the Leybold document. More detail is available in a nice article by P. Signell and V. Ross

A new description of the Cavendish experiment has been provided by the manufacturers of the equipment manufacture, Leybold. You should study this before coming to lab. Copies will be available in lab if you forget to bring a print-out.

This apparatus is extremely sensitive. It has a very high \( Q \) and once a vibration is excited it takes a very long time for it to damp out. You must be very careful to not bump or touch the table on which the apparatus is resting or to create any shock waves by allowing the door to slam when closing. You must also take care to not move unnecessarily in room while data taking is in progress. The mirrors should be aligned and in data taking position and must not be touched.

There are four stations with torsion balances and lasers set-up. When you arrive please enter and walk softly and sit at a table adjacent to the table on which the torsion balance and laser are resting. There should be no more than four students per torsion balance. Figures 3-5 show one of the torsion balances as set-up in the Phy231 laboratory. You will find the apparatus as shown: the large masses turned so that they are about 3-4 mm (about 1/8 inch) from the glass face of the enclosure with the suspended small masses mirror.
Fig. 1 Schematic of Cavendish apparatus

Fig. 2 Example of measured angle

Fig. 3 Apparatus on Table.

Fig. 4 View showing large masses and the torsion pendulum enclosure.

Fig. 5 View from side as you approach it to rotate the large masses to induce oscillations.
Procedure
Each group should verify that the basic measuring tools are available: Stop watch, ruler or tape measure, Graph paper in place on wall and original spot position. The first step is to verify the stability by observing the zero-point fluctuations for about 10 minutes, less if it is very stable. This establishes the stable equilibrium point and should be noted on the graph paper attached to the wall. Predict in which direction the light spot on the wall will move when the masses are rotated. Choose one person to rotate the masses. That person should approach the apparatus along the plane of the enclosure as viewed in figure 5. Rotate the masses using both hands and apply force near the bottom of the grey posts that support the large masses. Start the clock and observe the light spot on the wall as it begins to move. Is it moving in the expected direction? As the spot moves make measurements displacement measurements should be made every 20 to 30 seconds. Plotting the displacement from equilibrium versus time should result in a damped oscillation curve as shown in Figure 5 of the Leybold description. Continue measuring until you have observed at least four complete cycles. Before leaving measure the distance from the torsion pendulum to wall and the equilibrium point you marked on your graph paper. You will need the large tape measure for this. Do not forget to correct for sag of the tape.