EXPERIMENT 1

MEASUREMENT OF g USING A SIMPLE PENDULUM
In this first experiment we will make use of the simple pendulum in order to obtain a value for, g, the acceleration of gravity. The simple pendulum, as many of you will remember, consists of a small mass, a bob, suspended from a chord or string such that the length from point of suspension, pivot point, to the center of mass of the bob is L. The bob will, when displaced by a small angle θ, execute nearly simple harmonic motion. It can be shown that in this case, the period depends only on the length L and the acceleration of gravity according to $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$. We will exploit this simple dependence of period on L and g in order to obtain a measurement of g in the laboratory.

THEORETICAL BACKGROUND
The following web site provides the background theoretical information you need in order to understand the simple pendulum (http://www.gmi.edu/~drussell/Demos/Pendulum/Pendula.html). Any mechanics book that you used in your mechanics course will also have a detailed explanation of the simple pendulum and you should refer to one or more of these to develop the background needed for writing a complete lab report.

APPARATUS
The apparatus used is shown in the figure below.

As the bob is displaced to larger and larger angles and released, the pendulum no longer undergoes simple harmonic motion and the period is no longer constant. Our goal is to design an experiment that allows us to determine the value of the period, $T$, in the region where it depends only on L and g and from that value of $T$, obtain a measurement of g.

MEASUREMENTS
You will need to measure the length from the point where the string is attached, the pivot point, to the center of mass of the bob. Two bobs of different material are available. You should make one measurement of the period under identical conditions with each bob and determine if the period depends on the mass of the bob. Tools at your disposal for making measurements are a straight
edge ruler calibrated in metric units and caliper and a stopwatch. The caliper can be used to
determine, with reasonable accuracy, the diameter of the bob, while the ruler can be used to
determine the length of the supporting string from the top of the ball to its pivot point. From the
two measurements and your estimated errors, you will be able to obtain a measurement of length
from pivot point to the center of mass and its uncertainty, \( L \pm \Delta L \). The stop watch will determine
the time for the bob to undergo a number of complete cycles.

The next step will be to excite motion by extending the bob, releasing it and using your timers in
order to measure the period of motion. At this point one needs to measure an angle and the period
since you want to establish the dependence of period on angle. The angle can be measured with a
protractor, but since we will need many measurements at very small angles, this could introduce
more error than desirable. We need therefore to find a more precise way to determine the angle.
If we measure the distance, \( x \), through which the bob is displaced and use the definition of the sin
of the angle we can calculate the value of \( \theta \). The error on this measurement will be a combination
of the errors on the two length measurements.

Before beginning your measurement of the period as a function of the displacement angle you
need to establish the precision of each of the measured quantities: length of string from pivot point
to top of bob; radius of bob; and time measured on stop watch. The error on length and radius can
be established by the method discussed in lecture 2, see posted lecture notes. The time
measurements have random fluctuations in making the starting and stopping the clock coincident
with the bob reaching its maximum amplitude. To establish the distribution you need to make 50
measurements of a single period for each of the two bobs provided. You also will make 10-
measurements of the time for 10 periods for each of the bobs and divide each set by 10 to obtain
the average time for one period.

Your final data set will be represented by the measured period as a function of the angle \( \theta \). You
will therefore have a table in which for each angle you set, there is a value of the measured period.
You will need to make many measurements at very small angles in order to be able to extrapolate
to \( \theta = 0 \). Having established this you next need to make measurements at increasing angles in
order to determine how (functional dependence) and where (what value of \( \theta \)) it is no longer
appropriate to use the small angle approximation.

**DATA ANALYSIS**
You will need to make a histogram of the 50 measurements and also of the ten average
measurements in order to establish a distribution of time measurements from which an estimate of
\( \Delta t \) can be made. You will need to determine the bin size to use for your histogram. Your least
count is 0.01 s so this is the smallest the bins can be. If the distribution is very broad you may
need to bin in units of several least counts in order to have a significant number of events in each
bin. You should look at the distribution for each ball and if they are similar you may decide to
combine them into one distribution. That is a single period distribution with 100 entries and a
distribution of periods obtained by averaging 10 periods. Estimate the most probable period, \( T \), for
each distribution and also the \( \pm \Delta T \) about the most probable vale that would include about 2/3 of
the entries.
At this point the experimenter is ready to analysis the data from which $g$ will be determined. First task is to make a distribution of $T \pm \Delta T$ vs. $\theta$ and ask yourself what behavior one expects to observe in such a plot. The $\Delta T$ are to be determined by combining errors using the prescription described in section 3.11 of Taylor. The errors $\pm \Delta T$ are shown on the plot by bars extending $\Delta T$ above and below the plotted value of $T$. You can extrapolate the $T(\theta)$ plot to $\theta = 0$ to determine the $T$ to be used for the calculation of $g$. Alternately (is it better?) you could average a number of $T(\theta)$ to get an average $T$ to be used for the calculation of $g$. You will report a value of the acceleration of gravity with your experimental error, $g \pm \Delta g$. You should discuss at what value of displacement, $\theta$, is the simple pendulum approximation no longer valid. You also will report the period and error of the period obtained with bobs of different mass when the initial conditions $(t=0)$ are identical.